STAT 2593

Lecture 004 - Measures of Variability

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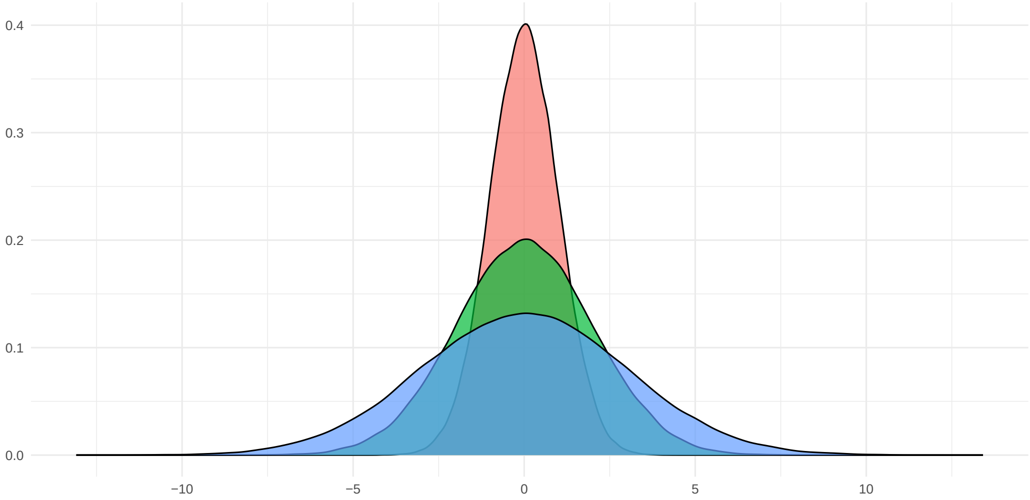
Measures of Variability

Learning Objectives

1. Understand and interpret the range, variance (and standard deviation), and percentiles

2. Understand and interpret boxplots

What does the Location Miss?



When some data are more *spread out* than others, we say that they have higher **variability**.

There is less concentration around the measures of location.

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  - The square root of the sample variance is called the **standard deviation**, $s = \sqrt{s^2}$.

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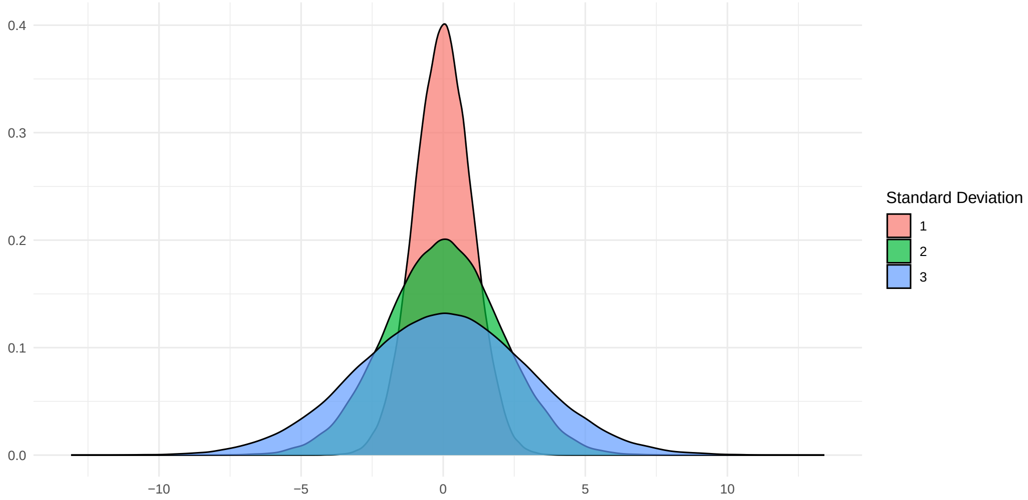
  - This can be useful for unit conversions.

What does the Location Miss? (Variation!)



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- We call the 25th percentile Q1, and the 75th percentile Q3.
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  - These can be computed as the median of the lower and upper half of the data.

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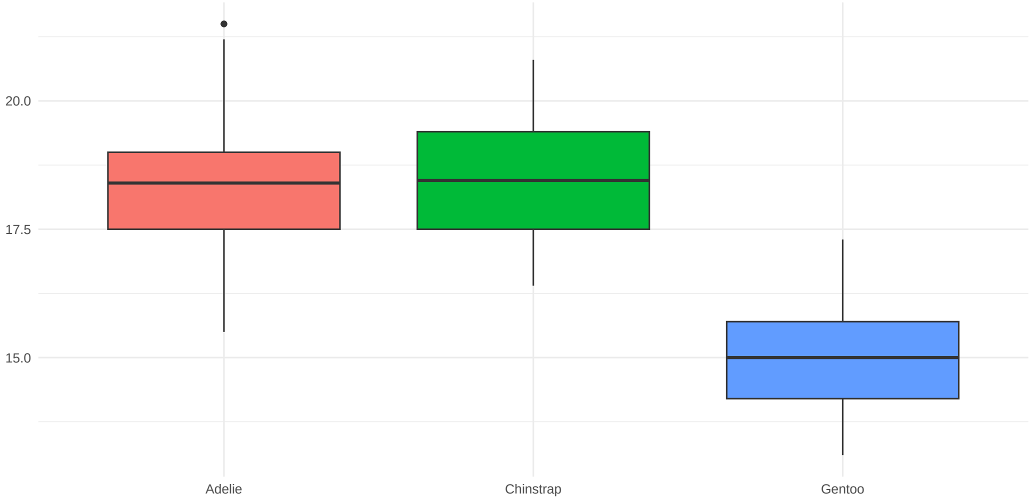
- If we list min, $Q1$, median, $Q3$, max for data, this is the **five number summary**.
  - We can display the five number summary using a **box plot**

Boxplots



Summary

- Location does not capture all the nuance of a particular distribution.
- It is important to consider the spread, or variability as well.
- The range, standard deviation, and variance are all common methods for measuring variability.
- Medians can be generalized to arbitrary values, called percentiles.
- Percentiles are used to form the IQR and the five number summary.
- The five number summary can be graphically represented through box plots.